Models of the cutting edge geometry of medical needles with applications to needle design

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1. Introduction

Medical needles are the most commonly used medical devices and are widely used in minimally invasive percutaneous procedures such as injection, regional anesthesia, blood sampling, biopsy and brachytherapy. These procedures require inserting a needle to a target inside the body for either drug delivery or tissue sample removal. Brachytherapy is a medical procedure to place radioactive seeds inside or near the tumor using a needle for cancer treatment. The success of this procedure depends on the accurate placement of the seeds. However, inserting a needle deforms the tissue and displaces the target causing placement errors [1,2]. Substantial practice is needed to avoid placement errors by compensating for tissue deformation and target displacement caused by the needle tip force and friction. It has been shown by Alterovitz [2–4] that without preoperative planning placement errors may be as large as 26% of the prostate diameter resulting in damage to healthy tissue and failure to kill cancerous cells. A practical approach to reducing the placement error is to minimize the needle insertion force. Thus, a needle that creates lower insertion forces and smaller displacements of the target is critical to the success of the procedure. Additionally, the studies by Egekvist et al. [5,6] and Arendt-Nielsen et al. [7] have shown that the needle insertion force is positively correlated with the frequency and intensity of pain and trauma. For example, the insertion force is significantly higher for 27G needle insertions compared to 30G needle insertions. Correspondingly, 53% of insertions with the 27G needles caused pain and 39% of insertions with 30G needles caused pain. In [8], the bluntness of the needle tip has been found to be related to the increased pain in subcutaneous injections. Thus, needles with lower insertion forces are important to reduce the pain and trauma experienced by the patients during the procedure.

Needle insertion is essentially a tissue cutting process. The force of tissue cutting by a needle greatly depends on its cutting edge geometry. Prior to studying the cutting edge geometry of a needle, one may begin by examining a blade with a straight cutting edge that is defined by its included angle θ and the cutting edge radius also referred to as the “roundness” of the cutting edge, as illustrated in Fig. 1(a). Maximal knife sharpness is obtained with a small included angle and a small radius for the cutting edge [9]. The knives with a small included angle, such as surgical scalpels and straight razors, are extremely sharp but fragile. On the contrary, knives that require a tough edge usually have a large included angle. Typical values of the included angle are about 15° for razor blades and veneer cutting knives, 20–30° for microtome knives and 30–40° for kitchen knives [9]. Typical radii of cutting edges are 5 μm for a...
scapel and 17 μm for a new safety razor blade (34 μm when worn) [9]. Another important factor in cutting is the inclination angle \( \alpha \), which describes the angle of the cutting edge relative to the cutting direction, as illustrated in Fig. 1(b). It has been shown by Atkins [10–12] that the cutting force exerted by a sharp object depends on the slice/push ratio \( k \) given by \( k = \text{speed parallel to cutting edge/speed perpendicular to the cutting edge} \). An increase in the slice/push ratio reduces the cutting force. When the cutting edge is inclined at an inclination angle \( \alpha \), the slice/push ratio is given by \( k = \tan \alpha \). An increase in the inclination angle increases \( k \), leading to a lower cutting force. For a general cutting process, three other basic angles are important: the rake angle \( (\zeta) \), the clearance/reief angle \( (\zeta) \), and the setting angle \( (\beta) \), as shown in Fig. 1(c). The cutting surface of the blade is the rake face. The rake angle is the angle of the rake face with respect to the perpendicular to the cut surface. The clearance/reief angle is the angle between the second face of the blade and the cut surface, and the rake surface. Hence, \( \theta + \alpha + \zeta = 90^\circ \). It can be seen that the rake angle and clearance/reief angle depend on the setting angle. Tissue cutting by a scalpel or a needle is a process where the tool cuts by penetration into the tissue. This type of cutting has a setting angle \( \beta = 0^\circ \), as shown in Fig. 1(d), where both the edge bevels act as rake faces, and \( \theta + \alpha_2 + \zeta = 180^\circ \). A symmetric blade has \( \alpha_1 = \alpha_2 \). For a blade with only one side beveled and the other side remaining flat, \( \alpha_2 = 90^\circ \), hence \( \theta + \alpha_1 = 90^\circ \).

Compared to a straight blade, a needle usually has more complicated cutting edges. For biopsy and brachytherapy, the needle often consists of an outer cannula (hollow needle) and an inner styllet (solid needle) [13]. Currently, there exist a wide variety of needle tip geometries. The simplest needle tip geometry is the one-plane bevel tip. A multi-plane needle is formed by planes oriented symmetrically or asymmetrically. Commercially, the symmetric three-plane styllet is called a trocar. Examples of needle tip geometries are illustrated in Fig. 2.

Surprisingly little research has been carried out related to the modeling of the needle cutting edge geometry and to investigate how the cutting edge geometry influences the penetration force of a needle. Studies have been conducted to investigate the effects of needle tip geometry on the penetration force and needle sharpness [14–20]. In these works needle tip geometry was defined only by the bevel angle, the angle between the bevel ground surface and the needle longitudinal axis, which does not directly reveal any specific information about the cutting edges. The needle sharpness was judged by the penetration force encountered during passing through tissue. However, little is known about how sharpeness and penetration force are directly influenced by tip geometry. Moore et al. [21–23] have used the customary cutting tool geometry approach widely used in metal cutting theory to model the hollow biopsy needle tip. The emphasis of their work was on the determination of the variation of the inclination and rake angles along the needle cutting edge. For a biopsy cannula, where the cutting edge is generated by the intersection of the needle side surface and the bevel plane (rake face), one rake angle together with the inclination angle and edge radius properly describes the cutting edge. However, in case of a multi-plane needle styllet where the needle cutting edge is formed by the intersection of two bevel surfaces, one rake angle together with the inclination angle and edge radius does not adequately represent the cutting edge geometry. Until now, no work has been reported on modeling the cutting edge of a needle styllet.

In this paper, the cutting edge geometry of a needle is defined by the included angle, the inclination angle, and the cutting edge radius, as is defined for a blade. The radius of a cutting edge depends on the manufacturing processes including the machining, deburring, and coating processes, while the included angle and inclination angle of a needle's cutting edge are determined by its tip geometry. From the design point of view, smaller included angle and larger inclination angle are desirable for reducing the penetration forces during needle insertion. However, the included angle and inclination angle of a needle's cutting edge are not explicitly defined. In the absence of closed form expressions for the included angle and inclination angle the only way to pinpoint the best needle tip design is via repeated experiments and simulations. Consequently, making a design decision about which needle type will result in minimal penetration and side/lateral forces becomes difficult.
This work addresses the above-mentioned issues in the context of the needle tip geometry by obtaining general expressions for the included angle and inclination angle of the needle cutting edge in Section 2. In Section 3 specific expressions are derived and used to examine asymmetric needles. At the same time, since asymmetric needles suffer from excessive deflection and poor targeting accuracy, Section 2 examines asymmetric needles. A novel needle tip design, a three-cylindrical-surface tip, is proposed while maintaining the advantage of lower deflection provided by symmetry. Therefore, this new design has significant potential for reducing patient trauma as well as target accuracy during needle insertion. Finally, Section 4 summarizes this work. The findings of this research will assist medical needle manufacturers in developing novel needles with lower penetration force to minimize pain and trauma. New biopsy needles will increase the quality of tissue samples and improve the accuracy of diagnosis. Needles with lower included and larger inclination angles will reduce the penetration force and placement errors in brachytherapy.

2. General mathematical model for included and inclination angles

In general, a needle tip is generated by planes or curved surfaces intersecting each other. Fig. 3 illustrates a needle tip formed by the intersection of two arbitrary surfaces \( F_1 \) and \( F_2 \) and of the needle body. A Cartesian coordinate system with the \( z \)-axis coinciding with the needle longitudinal axis and \( x \)-axis passing through the lowest point of the needle tip profile is defined. The cutting edge is the intersection between these two surfaces, and it can be mathematically given by

\[
\begin{align*}
F_1(x,y,z) = 0 \\
F_2(x,y,z) = 0
\end{align*}
\]

(1)

The normal vector at a point \((x_0,y_0,z_0)\) on a surface \(F(x,y,z)=0\) is, in turn, given by

\[
N = \begin{bmatrix} F_1(x_0,y_0,z_0) & F_2(x_0,y_0,z_0) & F_z(x_0,y_0,z_0) \end{bmatrix}
\]

(2)

where \(F_x = \partial F/\partial x\), \(F_y = \partial F/\partial y\), and \(F_z = \partial F/\partial z\) are partial derivatives.

At a point \(M_0(x_0,y_0,z_0)\) on the cutting edge, the normal vector to surface \(F_1\) is \(N_1 = \begin{bmatrix} (F_1,x)_{M_0} & (F_1,y)_{M_0} & (F_1,z)_{M_0} \end{bmatrix}\) and the normal vector to surface \(F_2\) is \(N_2 = \begin{bmatrix} (F_2,x)_{M_0} & (F_2,y)_{M_0} & (F_2,z)_{M_0} \end{bmatrix}\), as illustrated in Fig. 3. The included angle \(\theta\) is the angle between two surfaces \(F_1\) and \(F_2\). The dot product can be used to determine this angle. Given two normal vectors \(N_1\) and \(N_2\), the angle between the vectors \(N_1\) and \(N_2\) at \(M_0(x_0,y_0,z_0)\) is \(\pi - \theta\) and can be computed from

\[
\cos(\pi - \theta) = \frac{N_1 \cdot N_2}{|N_1||N_2|}
\]

(3)

and is explicitly given by

\[
\theta = \pi - \arccos \left( \frac{(F_1,x)_{M_0}(F_2,x)_{M_0} + (F_1,y)_{M_0}(F_2,y)_{M_0} + (F_1,z)_{M_0}(F_2,z)_{M_0}}{(F_1,x)_{M_0}^2 + (F_1,y)_{M_0}^2 + (F_1,z)_{M_0}^2 (F_2,x)_{M_0}^2 + (F_2,y)_{M_0}^2 + (F_2,z)_{M_0}^2)} \right)
\]

(4)

For percutaneous procedures, the penetration direction vector coincides with the needle longitudinal axis and can be expressed as \(v = [0 \ 0 \ 1]\), while the tangent vector to the cutting edge can be given by the cross product of vectors \(N_1\) and \(N_2\) as

\[
t = N_1 \times N_2 = \begin{bmatrix} i & j & k \\
(F_1,x)_{M_0} & (F_1,y)_{M_0} & (F_1,z)_{M_0} \\
(F_2,x)_{M_0} & (F_2,y)_{M_0} & (F_2,z)_{M_0} \end{bmatrix}
\]

(5)

The inclination angle \(\lambda\) is the angle between the vector \(t\) and the \(xy\)-plane (plane with normal vector \(v\)). It can be given by

\[
\lambda = \arcsin \left( \frac{|t \cdot v|}{|t||v|} \right)
\]

(6)

Eqs. (4) and (6) describe the included angle and inclination angle along the cutting edge formed by the intersection of two general surfaces.

3. Specific mathematical model for included and inclination angles

In this section, expressions will be given for specific needle tip geometries. Needle tips generated by planar surfaces will be evaluated first. The mathematical models will provide valuable information regarding the included angle and inclination angle of the needle cutting edge. The results will help in identifying the drawbacks of the planar needle and guide the design of new needles. Needle tips generated by curved surfaces will then be investigated. The benefits obtained from curved surfaces will also be described.

3.1. One-plane bevel tip needle

The bevel tip needle is made by one-plane being ground at a specific bevel angle \(\zeta\), where the leading edge of the needle tip acts as the cutting edge, as marked in Fig. 4. The angular position of a point on the cutting edge is defined by \(\gamma\) in the coordinate system, as shown in Fig. 4(a). The parametric equation of the cutting edge for a bevel tip needle with a radius of \(r\) is given by

\[
\begin{align*}
x &= r \cos \gamma \\
y &= r \sin \gamma \\
z &= r(1 - \cos \gamma) \cot \zeta
\end{align*}
\]

(7)

The angle between the needle side surface and the bevel plane is the included angle \(\theta\). From Eq. (2), the unit normal vector to the
needle side surface at a point on the cutting edge is \( c = [\cos \gamma \sin \gamma \ 0] \). The unit normal vector to the bevel plane is \( n = [\cos \xi \ 0 \ \sin \xi] \). From Eq. (3), one obtains

\[
\cos(\pi - \theta) = \frac{\mathbf{b} \cdot \mathbf{c}}{||\mathbf{b}|| ||\mathbf{c}||} = \cos \xi \cos \gamma \tag{8}
\]

The included angle \( \theta \) for a bevel tip needle is then given by

\[
\theta = \pi - \arccos (\cos \xi \cos \gamma) \quad 0 < \xi \leq \frac{\pi}{2} \tag{9}
\]

The equation for \( \theta \) of a bevel tip needle can be obtained and the result agrees with that by Moore [21].

\[
\hat{\lambda}(\xi, \gamma) = \arcsin \frac{|\cot \xi \sin \gamma|}{\sqrt{1 + \cot^2 \xi \sin^2 \gamma}} \quad 0 < \xi \leq \frac{\pi}{2} \tag{10}
\]

3.1.1 Results and discussion for bevel tip needles

The included angle \( \theta \) for needles with bevel angles \( \xi = 15^\circ, 30^\circ, 45^\circ \), and \( 60^\circ \) is plotted in Fig. 5(a). It is clear from the results that the bevel tip needle is symmetric about the \( xz \)-plane. Fig. 5(a) shows that \( \theta \) increases with the increase in \( \xi \). For a given \( \xi \), the minimum \( \theta \) is equal to \( \xi \) at the needle tip point (\( \gamma = 180^\circ \)). Fig. 5(b) plots the inclination angle \( \hat{\lambda} \) for the needles with \( \xi = 15^\circ, 30^\circ, 45^\circ \), and \( 60^\circ \).

At the tip point of the needle, \( \hat{\lambda} \) is equal to 0° regardless of \( \xi \), which is the worst possible cutting configuration [21,24]. Smaller \( \xi \) leads to larger inclination angles with a maximum equal to \( \hat{\lambda} = 90^\circ - \xi \) at \( \gamma = 90^\circ \) and \( \gamma = 270^\circ \).

At the tip point of the needle, i.e., at \( \gamma = 180^\circ \), the cutting edge has \( \theta = \xi \), the smallest possible included angle; however, the inclination angle is 0°. At \( \gamma = 90^\circ \) and \( \gamma = 270^\circ \), the cutting edge has \( \lambda = 90^\circ - \xi \), the largest possible inclination angle, however, the cutting edge has \( \theta = 90^\circ \), a very large included angle. Remembering that lower included angles and larger inclination angles are desirable for reducing needle penetration forces, the cutting edge of the bevel tip needle has an undesirable geometry. A larger \( \xi \) increases \( \theta \) and reduces \( \lambda \), thereby creating a higher penetration force during needle insertion [25]. Large penetration forces not only increase the patient pain and trauma but also cause needle placement errors, resulting in treatment failure. The tip geometry can be improved by using multi-plane needles, as will be discussed in the next section.

3.2 Asymmetric three-plane needle

Multi-plane needles are intended to obtain a desirable cutting edge configuration, i.e., a smaller included angle and a larger inclination angle. Among multi-plane needles, three-plane needles are widely used. For an asymmetric three-plane needle, the tip is generated by a primary bevel grind followed by two secondary bevel grinds. Fig. 6 illustrates an asymmetric three-plane needle formed by a primary bevel plane \( F_1 \) with a bevel angle of \( \xi_1 \) and two secondary bevel planes \( F_2 \) and \( F_3 \) with a bevel angle of \( \xi_2 \). The secondary bevel plane \( F_2 \) is obtained by rotating

![Fig. 4. Illustration of the bevel tip needle.](image1)

![Fig. 5. (a) included angle \( \theta \) and (b) inclination angle \( \hat{\lambda} \) for bevel tip needles.](image2)

![Fig. 6. Illustration of an asymmetric three-plane needle.](image3)
the primary bevel plane about the z-axis by a counterclockwise angle of $\phi$ followed by translating the interim plane along its normal direction by a distance $d$. This transformation is illustrated in Fig. 6. The secondary bevel plane $F_2$ is similarly obtained by rotating the primary bevel plane about the z-axis by a clockwise angle of $\phi$ followed by translating the interim plane along its normal direction by a distance $d$. Note that the intersection of the needle side surface and of the primary bevel plane is an ellipse with center $P_1$ and vertex $P$. The primary plane of $F_2$ passes through $P_1$ and has the same normal vector as $F_2$. The intersection of the primary bevel plane and secondary bevel planes forms the tip point. The location of this tip point may vary from $P$ to $P_1$, depending on $d$. When $d=0$ the tip point coincides with $P_1$; At a certain value of $d$, the tip point will coincide with $P$.

Among commercially available needles, the lancet tip and back bevel tip needles can be categorized as asymmetric three-plane needles. Both needles are geometrically asymmetric, but they are symmetric with respect to the xz-plane, as shown in Fig. 6. The secondary bevel plane divides the cutting edge into two sections. Section 1 is the cutting edge “PM” formed by the secondary bevel plane, while Section 2 is the remaining part of the bevel tip needle cutting edge, as marked in Fig. 7. The location of $M$ determines the transition position between the two sections and will be derived later. Note that “PM” is solely the cutting edge when the angular position $\gamma_M$ of point $M$ is less than 90°. For a lancet tip needle, the cutting edge “curve PM” is the intersection of the needle side surface and secondary bevel plane. For a back bevel tip needle, the cutting edge “line PM” is the intersection of the primary bevel plane and of the secondary bevel plane. Whether the cutting edge is “curve PM” or “line PM” depends on the angle $\phi$. When $\phi < 90°$, the included angle of “curve PM” is smaller than that of “line PM”, which makes “curve PM” the cutting edge. When $\phi > 90°$, the included angle of “line PM” is smaller than that of “curve PM”, which makes “line PM” the cutting edge.

The unit normal vector to the needle side surface at a point on the cutting edge is $\mathbf{n} = \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \end{bmatrix}$. The unit normal vector to the primary bevel plane is $\mathbf{b}_1 = \begin{bmatrix} \cos \xi_1 & 0 & \sin \xi_1 \end{bmatrix}$. The unit normal to the secondary bevel plane can be given by

$$b_2 = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \xi_2 \\ \sin \xi_2 \\ \sin \xi_2 \end{bmatrix}$$

(11)

with the normal vectors to the surfaces that form the cutting edges defined, one can obtain the included and inclination angles from Eqs. (4)-(6).

3.2.1. Lancet tip needle

The included angle of the cutting edge “curve PM” for a lancet tip needle is the angle between the needle side surface and the secondary bevel plane. Given the normal vectors $c$ and $b_2$, the equation for the included angle $\theta$ of a lancet tip needle is obtained using Eq. (4) as

$$\theta(\xi_2, \gamma; \phi) = \pi - \arccos (\cos \xi_2 \cos \gamma - \sin \xi_2 \sin \gamma)$$

$$0 < \xi_2 \leq \frac{\pi}{2}$$

(12)

The tangent vector of the cutting edge “curve PM” is the cross product of the normal vectors $c$ and $b_2$ and can be obtained using Eq. (5) $t = \begin{bmatrix} \sin \gamma & -\cos \gamma & -\cos \xi_2 \sin \gamma \end{bmatrix}$. The cutting direction vector is $\nu = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$. From Eq. (6), the inclination angle $\lambda$ for the lancet tip needle can be obtained and the result agrees with that by Moore [21]

$$\lambda(\xi_2, \gamma; \phi) = \arcsin \left( \frac{|\cos \xi_2 \sin (\gamma - \phi)|}{\sqrt{1 + \cos^2 \xi_2 \sin^2 (\gamma - \phi)}} \right)$$

$$0 < \xi_2 \leq \frac{\pi}{2}$$

(13)

By comparing the included angle and inclination angle expressions for the bevel tip needle (Eqs. (9) and (10)) with those of the lancet tip needle (Eqs. (12) and (13)), it can be seen that the included angle and inclination angle for a lancet tip needle can be obtained from those for a bevel tip needle with a phase shift of $\phi$.

3.2.2. Back bevel tip needle

The included angle $\theta$ of the cutting edge “line PM” for a back bevel tip needle is the angle between the primary and secondary bevel planes. Given the normal vectors $b_1$ and $b_2$, the equation for the included angle $\theta$ for a back bevel tip needle can be obtained from Eq. (4) as

$$\theta(\xi_1, \phi) = \pi - \arccos (\cos \xi_1 \cos \phi + \sin \xi_1 \sin \xi_2)$$

$$0 < \xi_1 \leq \frac{\pi}{2}$$

(14)

The tangent vector of the cutting edge “line PM” is the cross product of the normal vectors $b_1$ and $b_2$ and can be obtained using Eq. (5):

$$t = \begin{bmatrix} -\sin \xi_1 \cos \xi_2 \sin \phi \sin \xi_1 \cos \xi_2 \cos \phi - \cos \xi_1 \sin \xi_2 \\ \cos \xi_1 \cos \xi_2 \sin \phi + \cos \xi_1 \sin \xi_2 \cos \xi_2 \sin \xi_2 \end{bmatrix}$$

(15)

From Eq. (6), the inclination angle is therefore obtained as

$$\sin \lambda = \frac{|\cos \xi_2 \cos \xi_1 |}{\sqrt{\sin^2 \xi_1 \cos^2 \xi_2 \sin^2 \phi + (\sin \xi_1 \cos \xi_2 \cos \phi - \cos \xi_1 \sin \xi_2 \cos \xi_2 \sin \phi)^2 + (\cos \xi_1 \cos \xi_2 \sin \phi)^2}}$$

(16)

To simply the equation for $\lambda$, the inclination angle is expressed in terms of the tangent of $\lambda$, which gives

$$\tan \lambda = \frac{|\sin \phi|}{\sqrt{\tan^2 \xi_1 + \tan^2 \xi_2 - 2 \tan \xi_1 \tan \xi_2 \cos \phi}}$$

(17)
Finally, the inclination angle $\lambda$ for a back bevel needle is given by

$$\lambda(\xi_1, \xi_2, \phi) = \arctan \left( \frac{\sin \phi}{\sqrt{\tan^2 \xi_1 + \tan^2 \xi_2 - 2\tan \xi_1 \tan \xi_2 \cos \phi}} \right)$$

$$0 < \xi_{1,2} \leq \frac{\pi}{2}$$

(18)

3.2.3. Determination of the transition point $M$

The location of point $M$ determines the transition position between two sections of the cutting edge. More importantly, point $M$ determines the length of the cutting edge formed by the secondary bevel plane, which influences the shape of the incision hole and the penetration force [26]. The transition point for a lancet tip cannula has been derived by Wang et al. [27]. The same approach is used here for the derivation of the location of point $M$.

The tangent direction of “line $PM$” has been given by Eq. (15). The slope of “line $PM$” in the $xy$-plane can be expressed as

$$m = \frac{\sin \xi_1 \cos \xi_2 \cos \phi - \cos \xi_1 \sin \xi_2}{-\sin \xi_1 \cos \xi_2 \sin \phi} = \cot \xi_1 \tan \xi_2 - \cos \phi$$

$$\sin \phi$$

(19)

with the point $P(-r, 0)$ and the slope $m$, the coordinates of point $M$ in the $xy$-plane can be solved and are given by $M((r-m^2)/\sqrt{1+m^2}, 2mr/(1+m^2))$. The angular position $\gamma_M$ of point $M$ can be solved from

$$r \cos \gamma_M = \frac{r-m^2}{1+m^2}$$

(20)

yielding

$$\gamma_M = \arccos \left( \frac{1-m^2}{1+m^2} \right)$$

(21)

3.2.4. Results and discussion for asymmetric three-plane needles

As previously discussed, when $\gamma_M > 90^\circ$, the secondary bevel plane divides the cutting edge into two sections. Section 1 is the cutting edge “$PM$” and Section 2 is the remaining part of the bevel tip needle cutting edge. The included and inclination angles for Section 2 remain the same as for the bevel tip needle. However, the included and inclination angles for Section 1 will be different and will be highlighted in different colors and markers in the following figures. Since both the lancet tip and the back bevel tip needles are symmetric about the $xz$-plane, only the results from $\gamma=90^\circ$ to $\gamma=180^\circ$ will be plotted. When $\gamma_M < 90^\circ$, “$PM$” is solely the cutting edge. The results from $\gamma=\gamma_M$ to $\gamma=180^\circ$ will be plotted.

For a lancet tip needle, $\phi$ needs to be smaller than $90^\circ$. Fig. 8 shows needles with a bevel tip: needle-1 and with lancet tips: needle-2, needle-3, and needle-4, at $\xi_1=15^\circ$, $\phi=45^\circ$, and $\xi_2=18^\circ$, $22.5^\circ$, $30^\circ$. It can be noticed that the length of the cutting edge “curve $PM$” increases with the decrease in the secondary bevel angle. Fig. 9 plots the included angle $\theta$ and the inclination angle $\lambda$ for these needles. In Fig. 9(b) the values of $\lambda$ are plotted from $\gamma_M$ to $\gamma=180^\circ$ for needles 2, 3 and 4 which corresponds to the cutting edge formed by the secondary bevel planes. It can be observed that the value of $\lambda$ at $\gamma=180^\circ$, i.e., the needle tip point, is significantly greater for these needles 2, 3 and 4 as compared to $\lambda$ at $\gamma=180^\circ$ for the bevel tip needle. Therefore, lancet tip needles are much better than the bevel tip needle as far as inclination angle is concerned. From a design point of view, Fig. 9(b) shows that higher inclination angles can be achieved with smaller secondary bevel angles. However, the included angle, shown in Fig. 9(a), is increased compared to the bevel tip needle. Slightly smaller included angles can be achieved with smaller secondary bevel angles, but the reduction is very limited. Therefore, the lancet tip needles increase the inclination angle at the cost of an increase in the included angle as well.

For a back bevel tip needle, $\phi$ needs to be larger than $90^\circ$. Back bevel tip needles with $\xi_1=15^\circ$, $\phi=135^\circ$, and $\xi_2=5^\circ$, $10^\circ$, $15^\circ$ are shown in Fig. 10. It can be seen that the cutting edge formed by the secondary bevel planes features a straight cutting edge with constant included and inclination angles. Like the lancet tip needle, the length of the cutting edge, “line $PM$” increases with the decrease in the secondary bevel angle. Fig. 11 plots the included angle $\theta$ and inclination angle $\lambda$ for these needles.
From Fig. 11(b), it can be noticed that the inclination angle has greatly increased with the back bevel cut. Higher inclination angles can be achieved with smaller secondary bevel angles. At the same time, Fig. 11(a) shows that the included angle of back bevel tip needle is significantly lower compared to that of the lancet tip needle.

By comparing Fig. 9 with Fig. 11, it can be seen that both the lancet tip needle and back bevel tip needle have large inclination angles, however, in terms of the included angle, the back bevel tip needle has a better configuration. Therefore, it is to be expected that the insertion forces for the back bevel tip needle will be lower than those for the lancet tip needle. This is in fact what was observed by Suzuki et al. [26] who showed that the penetration force by a back bevel tip needle is 40% lower than that by the lancet tip needle. For both lancet and back bevel tip needles, it is clear that the included angle, inclination angle, and cutting edge length depend on \( x_1, f, \) and \( x_2. \) The tip geometry may be optimized by varying the angles, which is not the scope of this paper. However the findings from the above-presented models create an opportunity for medical needle manufacturers to design lancet and back bevel needles with lower included angles and larger inclination angles.

An issue with asymmetric needles, including the bevel tip needle, lancet needle, and back bevel tip, is needle deflection during percutaneous procedures, because the tissue exerts higher forces on one side. Needle bending is one of the reasons for placement errors in needle insertion procedures. Preoperative planning is needed to avoid placement errors. A symmetric needle reduces the deflection during needle insertion. Therefore, mathematical models for symmetric needles will be given in the following sections.

### 3.3. Symmetric multi-plane needle

A symmetric multi-plane needle is formed by bevel planes oriented symmetrically. The cutting edge is the intersection between two bevel planes. This needle features straight cutting edges with a constant included angle and inclination angle given the number of bevel planes \( P \) and bevel angle \( \xi. \) A symmetric two-plane stylet has \( P = 2 \) and therefore has a single cutting edge. The number of cutting edges for a multi-plane stylet is equal to \( P \) for \( P > 2. \) Fig. 12 illustrates the cutting edges, bevel planes, and bevel angles for a symmetric three-plane needle, which is the most popular commercially available needle today.

The unit normal vector to the bevel plane formed by the cutting edges \( PB \) and \( PC, \) as shown in Fig. 12, is \( b_1 = \begin{bmatrix} \cos \xi & 0 & \sin \xi \end{bmatrix}. \) The unit normal vector to the bevel plane formed by the cutting edges of \( PC \) and \( PA \) can be given by

\[
\begin{bmatrix}
\cos \phi & -\sin \phi & 0 \\
\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos \xi & 0 \\
\sin \xi \\
0
\end{bmatrix}
= \begin{bmatrix}
\cos \xi \cos \phi & \cos \xi \sin \phi \\
\sin \xi \cos \phi & \sin \xi \sin \phi \\
0 & 1
\end{bmatrix}
\]

where \( \phi \) is the rotation angle about the z-axis between successive bevel planes and is therefore \( \phi = 2\pi/P, \) and \( P \) is an integer representing the number of planes.

---

**Fig. 10.** Back bevel tip needles with various secondary bevel angles.

**Fig. 11.** (a) included angle \( \theta \) and (b) inclination angle \( \lambda \) for back bevel tip needles.

**Fig. 12.** Illustration of a three-plane symmetric needle.
As before, with the normal vectors to the surfaces that form the cutting edges determined, the included and inclination angles can be obtained from Eqs. (4)–(6).

The included angle $\theta$ for a symmetric multi-plane needle is the angle between two successive bevel planes. With the normal vectors $b_1$ and $b_2$, the equation for $\theta$ for a multi-plane needle can be obtained from Eq. (4) as

$$\theta = \pi - \arccos \left( \cos^2 \xi \cos \phi + \sin^2 \xi \right)$$

$$\phi = \frac{2\pi}{P}$$

$$0 < \xi \leq \frac{\pi}{2}$$ (23)

The tangent vector of the cutting edge is the cross product of the normal vectors $b_1$ and $b_2$ and can be given by $t = \left[ -\sin \phi \cos \phi - 1 \sin \phi \cot \xi \right]$. The angle between the xy-plane and vector $t$ is the inclination angle $\lambda$. From Eq. (6), the inclination angle can be given by

$$\sin \lambda = \frac{\cot \xi \sin \phi}{\sqrt{2 - 2\cos \phi + \sin^2 \phi \cot^2 \xi}}$$ (24)

To simply the equation, the inclination angle can be expressed as

$$\tan \lambda = \cos \phi \cot \xi$$ (25)

Hence, equation for $\lambda$ of a symmetric multi-plane needle can finally be given by

$$\lambda(\xi, \phi) = \arctan \left( \cos \phi \cot \xi \right)$$

$$\phi = \frac{2\pi}{P}$$

$$0 < \xi \leq \frac{\pi}{2}$$ (26)

3.3.1. Results and discussion for symmetric multi-plane needles

Fig. 13(a) shows the included angle $\theta$ for needles with $P=2$, 3, 4, and 5. It can be seen that a smaller bevel angle $\xi$ leads to a smaller included angle $\theta$ where the minimal $\theta$ is limited to $(P-2)\pi/P$. The minimal $\theta$ for $P=2$, 3, and 4 is limited to $0^\circ$, $60^\circ$, and $90^\circ$, respectively. At the same time for a given $\xi$, the included angle increases with the increase in $P$. $P=2$ gives the minimal included angle, however, its inclination angle is equal to zero along the entire cutting edge. An increase in $P$ beyond 3 greatly increases the included angle, thereby making the needle less effective in cutting. As $P$ approaches infinity, $\theta$ approaches $\pi$ and the multi-plane needle approaches a conical needle. The inclination angle for needles with $P=2$, 3, 4, and 5 is shown in Fig. 13(b). The inclination angle is equal to zero along the entire cutting edge with $P=2$, regardless of $\xi$. Smaller $\xi$ leads to larger inclination angles with $P>2$. For a specific $\xi$, the inclination angle increases with the increase in $P$.

$P=3$ gives the best combination of included and inclination angles. Commercially, the symmetric three-plane needle is called a trocar. However, the results show that the included angle for a symmetric three-plane needle is always larger than $60^\circ$. For instance, when $\xi=15^\circ$, the included angle of the cutting edge for a three-plane needle is $66^\circ$. It is clear from the above results that the cutting edges of a multi-plane needle have a large included angle which is undesirable for lowering the insertion force. In order to achieve an included angle less than $60^\circ$, models for needles formed by curved surfaces will be presented in the next section.

3.4. Symmetric three-curved-surface needle

The majority of current needles are formed by using planar surfaces. The above-presented models have shown that the cutting edges of a symmetric three-plane needle have a large included angle. To address this problem, curved surfaces can be used to generate the needle tip. The most common curved surfaces are quadratic surfaces. Examples of quadratic surfaces include the cone, cylinder, elliptic cylinder, sphere, etc. Considering that the grinding wheel surface is a cylinder, mathematical models for needles formed by cylinders, as shown in Fig. 14, will be derived. The cutting edges are the intersections of these three surfaces.

Fig. 13. (a) included angle $\theta$ and (b) inclination angle $\lambda$ for symmetric multi-plane needles.

Fig. 14. Illustration of a needle formed by three cylinders, (a) front view showing the needle and one cylinder, and (b) plan view showing the needle and three cylinders.
cylinders. The included angle and inclination angle depend on the angle between the needle axis and the cylinder axis \( \xi \), the needle radius \( r \), and the cylinder radius \( R \), as shown in Fig. 14.

In order to describe the position and orientation of each cylinder, four coordinate systems are assigned, one to each cylinder (OXYZ to the needle, \( O_1X_1Y_1Z_1 \) to cylinder-1, \( O_2X_2Y_2Z_2 \) to cylinder-2, and \( O_3X_3Y_3Z_3 \) to cylinder-3). The coordinate systems are assigned in a way that their origins coincide with each other and the \( z \)-axis of each coordinate system coincides with the cylinder longitudinal axis. At this point one could express each cylinder in its native coordinate system by

\[
F(x, y, z) = x^2 + y^2 - R^2 = 0
\]  

where \( n = 1, 2, 3 \), and \( R \) is the radius of the cylinder.

The description of the cylinders with respect to the coordinate system OXYZ needs to be given in order to establish the needle tip geometry. \( O_1X_1Y_1Z_1 \) is obtained by rotating OXYZ around the \( x \)-axis by a counterclockwise angle of \( \xi \), \( O_2X_2Y_2Z_2 \) is obtained by rotating OXYZ around the \( z \)-axis by a counterclockwise angle of 120° followed by a rotation about the new \( x \)-axis by a counterclockwise angle of \( \xi \). \( O_3X_3Y_3Z_3 \) is similarly obtained by rotating OXYZ around the \( x \)-axis by a counterclockwise angle of 120° followed by a rotation about the new \( x \)-axis by a counterclockwise angle of \( \xi \). The corresponding transformations can be expressed as

\[
\begin{align*}
\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ \end{bmatrix} & = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \xi & \sin \xi \\ 0 & -\sin \xi & \cos \xi \\ \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \end{bmatrix} \\
\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ \end{bmatrix} & = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \xi & \sin \xi \\ 0 & -\sin \xi & \cos \xi \\ \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \end{bmatrix} \\
\begin{bmatrix} x_3 \\ y_3 \\ z_3 \\ \end{bmatrix} & = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \xi & \sin \xi \\ 0 & -\sin \xi & \cos \xi \\ \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \end{bmatrix}
\end{align*}
\]

The functions of the cylinders relative to the coordinate system OXYZ can then be given by

\[
F_1(x, y, z) = x^2 + (y\cos \xi + z\sin \xi)^2 - R^2 = 0 \quad (29a)
\]

\[
F_2(x, y, z) = \left( \frac{1}{2}x - \sqrt{\frac{3}{2}}y \right)^2 + \left( \frac{\sqrt{3}}{2}x - \frac{1}{2}y \right)\cos \xi - z\sin \xi \right)^2 - R^2 = 0 \quad (29b)
\]

\[
F_3(x, y, z) = \left( \frac{1}{2}x + \sqrt{\frac{3}{2}}y \right)^2 + \left( \frac{\sqrt{3}}{2}x + \frac{1}{2}y \right)\cos \xi + z\sin \xi \right)^2 - R^2 = 0 \quad (29c)
\]

A symmetric three-cylindrical-surface needle has three cutting edges which are the intersections of the three cylinders. Due to the symmetry of the needle tip, only the equations for one cutting edge and of the associated included and inclination angles need to be derived. It is evident from Fig. 14(b) that the cutting edge formed by the cylindrical surfaces \( F_2 \) and \( F_3 \) lies in the \( yz \)-plane and, therefore, \( x = 0 \). The equation for this cutting edge can then be obtained by setting \( x = 0 \) in Eqs. (29b) or (29c), i.e.:

\[
\left( \frac{\sqrt{3}}{2}y \right)^2 + \left( -\frac{1}{2}y\cos \xi + z\sin \xi \right)^2 - R^2 = 0
\]  

Expressing \( z \) explicitly as a function of \( y \), one obtains:

\[
z = \frac{y\cos \xi + \sqrt{-3y^2 + 4R^2}}{2\sin \xi}
\]  

\[0 \leq y \leq r\]

where \( r \) is the radius of the needle, \( R \) is the radius of the cylinder used to form the needle tip, and \( \xi \) is the angle between the needle axis and cylinder axis.

The normal vector to the surface \( F_2 \) at a point on the cutting edge is

\[
b_2 = \left[ \begin{array}{c} \frac{1}{2}x - \sqrt{\frac{3}{2}}y \\ \frac{\sqrt{3}}{2}x - \frac{1}{2}y \\ \cos \xi - \sin \xi \\ \end{array} \right] = \sqrt{\frac{3}{2}}x + \frac{1}{2}y \cos \xi - \sin \xi
\]

The normal vector to the surface \( F_3 \) at a point on the cutting edge is

\[
b_3 = \left[ \begin{array}{c} \frac{1}{2}x + \sqrt{\frac{3}{2}}y \\ \frac{\sqrt{3}}{2}x + \frac{1}{2}y \\ \cos \xi + \sin \xi \\ \end{array} \right] = \sqrt{\frac{3}{2}}x - \frac{1}{2}y \cos \xi + \sin \xi
\]

with the normal vectors \( b_2 \) and \( b_3 \), the equation for the included angle \( \theta \) at any point \( M_0(x_0, y_0, z_0) \) on the cutting edge can be given by

\[
\theta = \pi - \arccos \left( \frac{\|b_2 \cdot b_3\|}{\|b_2\| \|b_3\|} \right) = \arccos \left( \frac{\|F_2 \times F_3\|}{\|F_2\| \|F_3\|} \right)
\]

\[
\lambda = \arcsin \left( \frac{\|t \times v\|}{\|t\| \|v\|} \right) = \arcsin \left( \frac{|(b_2 \times b_3) \cdot v|}{\|b_2 \times b_3\| \|v\|} \right)
\]

3.4.1. Results and discussion for three-cylindrical-surface needles

An example of a three-cylindrical-surface needle is shown in Fig. 15. The position of the needle tip point corresponds to \( y = 0 \) and \( z = r \) is the end of the cutting edge.

Fig. 16 plots the included angle \( \theta \) and inclination angle \( \lambda \) for three-cylindrical-surface needles with \( r = 0.5 \text{ mm} \), \( R = 2, 3, 4 \text{ mm} \), and \( \xi = 10^\circ \). For comparison, the included and inclination angles for a three-plane needle with \( r = 0.5 \text{ mm} \) and \( \xi = 10^\circ \) are also plotted.

It can be seen from Fig. 16(a) that the included angle of a three-cylindrical-surface needle is smaller than that of a three-plane

![Fig. 15. An example of three-cylindrical-surface needle.](Image)
The included angle of a three-plane needle is constant and it is always larger than 60°. The included angle of a three-cylindrical-surface needle is equal to the included angle of a three-plane needle at the tip point \( y = 0 \), but it decreases from \( y = 0 \) to \( y = r \). The included angle increases with the increase in \( R \). As \( R \) approaches infinity, the three-cylindrical-surface needle degenerates to a three-plane needle. Thus, a small \( R \) is needed in order to achieve a small included angle. It is clear from the results that the needle generated by curved surfaces can achieve a much smaller included angle than a planar needle. Although the inclination angle of a three-cylindrical-surface needle decreases, the reduction in the included angle is twice as much as the reduction in the inclination angle. In addition, one can always decrease \( \zeta \) to achieve the desired included angle and inclination angle for a three-cylindrical-surface needle.

From the manufacturing point of view, this needle can be made using a convex grinding wheel, as illustrated in Fig. 17. The curvature of the grinding wheel surface corresponds to the cylinder radius \( R \) in the mathematical model. The angle \( \zeta \) can be controlled by adjusting the position and orientation of the needle relative to the grinding wheel.

4. Conclusions

This study has developed general and specific mathematical models for the included and inclination angles of the cutting edges for a variety of needles. The included angle and inclination angle of a one-plane needle were first investigated. The results show that it has a very undesirable configuration. The mathematical models for asymmetric three-plane needles were then derived. Specifically, the lancet tip needle and back bevel tip needle were investigated. It was shown that the back bevel tip needle can achieve a much smaller included angle than a lancet tip needle. In evaluating the included angle and inclination angle of a symmetric multi-plane needle, it was found that its minimal included angle obtained was limited. For a symmetric three-plane needle, commercially called a trocar needle, the included angle is always larger than 60°. To resolve this issue a novel symmetric three-cylindrical-surface needle was proposed and the respective mathematical models were formulated. The results demonstrated that the included angle of a three-cylindrical-surface needle has no limitation and is much smaller than that of a three-plane needle, which potentially lowers the penetration force during percutaneous procedures. Therefore, this newly proposed needle geometry combines the advantages of better targeting provided by the symmetry of the tip with reduced insertion forces due to a lower included angle and inclination angle than currently used symmetric three-plane needles.

This work provides a prerequisite for designing needle tip geometries. The mathematical models formulated in this work can serve as the basis for medical needle manufacturers to design optimized needles. Newly designed needles will benefit all percutaneous procedures. The pain and trauma experienced by the patients will be reduced. New biopsy needles will increase the quality of tissue samples and improve the accuracy of diagnosis. New brachytherapy needles will reduce the penetration force and placement errors.

Future work of this research includes optimizing the included and inclination angles of various needles based on the model. Newly designed needles with various included and inclination angles will be manufactured and the insertion/penetration force will be experimentally studied to determine the most efficient geometry.

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